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SEARCH OF EFFECTIVE ROTATION PATTERNS IN PRESENCE OF AUXILIARY INFORMATION IN SUCCESSIVE SAMPLING OVER TWO OCCASIONS

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ABSTRACT

In successive (rotation) sampling over two occasions with partial replacement of units at current (second) occasion, utilizing the information on an auxiliary character over both the occasions along with the information from previous occasion on study character, regression type estimators for estimating the population mean at current (second) occasion have been proposed. Behaviours of the proposed estimators have been studied. Proposed estimators have been compared with the sample mean estimator when there is no matching and the optimum estimator, which is a linear combination of the means of the matched and unmatched portion of the sample at the current (second) occasion. Optimum replacement policy is also discussed. Results have been demonstrated through empirical and pictorial means of elaboration.

Key words: Successive sampling, partial replacement, partial regression coefficient, regression type, bias, mean square error, optimum replacement policy.

1. Introduction

A change is an inherent behaviour of the nature. Some types of changes directly or indirectly affect the quality of living and surrounding of human beings. This requires continuous monitoring of the real life situation in hand. If situations required to be monitored are concerned with a very large group of individuals (population or universe), it is a difficult, time taking and costly affair. To meet these requirements, rotation (successive) sampling provides a strong statistical tool for generating the reliable estimates at different occasions.

Theory of rotation (successive) sampling appears to have started with the work of Jessen (1942). He pioneered using the entire information collected in the previous investigations (occasions). Further the theory of rotation (successive) sampling was extended by Patterson (1950), Rao and Graham (1964), Gupta

(1979), Das (1982), Chaturvedi and Tripathi (1983) and many others. Sen (1971) developed estimators for the population mean on the current occasion using information on two auxiliary variates available on previous occasion. Sen (1972, 1973) extended his work for several auxiliary variates. Singh and Singh (1991) and Singh and Singh (2001) used the auxiliary information on current occasion for estimating the current population mean in two occasions successive sampling. Singh (2003) extended their work for h-occasions successive sampling. Feng and Zou (1997) and Biradar and Singh (2001) used the auxiliary information on both the occasions for estimating the current mean in successive sampling. In many situations, information on an auxiliary variates may be readily available on the first as well as on the second occasion, for example, tonnage (or seat capacity) of each vehicle or ship is known in survey sampling of transportation, number of beds in different hospitals may be known in hospital surveys, number of polluting industries is known in environmental survey, nature of employment status, educational status, food availability & medical aids of a locality are well known in advance for estimating the various demographic parameters in demographic surveys. Many other branches in biological (life) sciences could be traced out to show the benefits of the present study. Utilizing the auxiliary information on both the occasions Singh (2005), Singh and Priyanka (2006, 2007, 2008) proposed chain type ratio, difference and regression type estimators for estimating the population mean at current (second) occasion in two-occasion rotation (successive) sampling.

The objective of present work is to propose two new estimators for estimating the population mean at current (second) occasion in two-occasion rotation (successive) sampling by utilizing the information on sample partial regression coefficients. Information on sample partial regression coefficients helps us in identifying whether the variation in the study character from occasion to occasion is due to the presence of auxiliary character or the change is due to the sensitivity of the character under study itself. The detail behaviours of the proposed estimators have been studied and conclusions have been drawn with the help of tabular and pictorial representation.

2. Proposed estimators

Let $U = (U_1, U_2, \ldots, U_N)$ be a finite population of size N, which has been sampled over two occasions, and the character under study be denoted by x(y) on the first (second) occasion respectively. It is assumed that information on an auxiliary variable z (with known population mean) is available on both the occasions. A simple random sample (without replacement) of n units is taken on the first occasion. A random sub-sample of $m = n\,\lambda$ units is retained (matched) for its use on the second occasion, while a fresh simple random sample (without replacement) of $u = (n - m) = n\,\mu$ units are drawn on the second occasion from the remaining non-sampled units of the population so that the sample size on the second occasion is also n. λ and μ ($\lambda + \mu = 1$) are the fractions of matched and fresh samples respectively.

The following notations have been considered for the further use:

$\bar{X}, \bar{Y}, \bar{Z}$: The population means of the variables x, y and z respectively.

$\bar{x}_n, \bar{z}_n, \bar{y}_m, \bar{x}_m, \bar{z}_m, \bar{y}_u, \bar{z}_u$: The sample means of the respective variates of the sample sizes shown in suffices.

$\rho_{yx}, \rho_{xz}, \rho_{yz}$: The correlation coefficients between the variates shown in suffices.

S_x^2, S_y^2, S_z^2: The population mean squares of x, y and z respectively.

$\beta_{yx.z}$ and $\beta_{yz.x}$: The population partial regression coefficients.

$b_{yx.z}$ and $b_{yz.x}$: The sample partial regression coefficients.

To estimate the population mean $\bar{Y}$ on the second (current) occasion, three different estimators T_{1u}, T_{1m} and T_{2m} are considered. The estimator T_{1u} is based on the sample of size u drawn afresh on the second occasion and estimators T_{1m} and T_{2m} are based on the sample of size m common with both the occasions. Estimators T_{1u}, T_{1m} and T_{2m} are formulated as:

$$T_{1u} = \bar{y}_u + b_{yz}(u)\left(\bar{Z} - \bar{z}_u\right), \quad T_{1m} = \bar{y}_m^* + b_{yx.z}(m)\left(\bar{x}_n - \bar{x}_m\right),$$
$$T_{2m} = \bar{y}_m^* + b_{yx.z}(m)\left(\bar{x}_n^* - \bar{x}_m^*\right)$$

where

$$\bar{y}_m^* = \bar{y}_m + b_{yz.x}(m)\left(\bar{Z} - \bar{z}_m\right), \quad \bar{x}_m^* = \bar{x}_m + b_{xz}(m)\left(\bar{Z} - \bar{z}_m\right),$$
$$\bar{x}_n^* = \bar{x}_n + b_{xz}(n)\left(\bar{Z} - \bar{z}_n\right).$$

For estimating the mean on each occasion estimator, T_{1u} is an appropriate choice, while for measuring the change from occasion to occasion estimators T_{1m} and T_{2m} are the best option under the given situation. To consider both the problems simultaneously some suitable combinations of the estimators T_{1u}, T_{1m} and T_{2m} are desired.

Motivated with the above discussion two different convex linear combinations T_{11} and T_{12} of the estimators (T_{1u}, T_{1m}) and (T_{1u}, T_{2m}) have been considered for estimating the population mean $\bar{Y}$ at current (second) occasion. T_{11} and T_{12} are defined as

$$T_{11} = \varphi_1 T_{1u} + \left(1 - \varphi_1\right) T_{1m} \tag{1}$$

and

$$T_{12} = \varphi_2 T_{1u} + \left(1-\varphi_2\right) T_{2m} \tag{2}$$

where φ_i (i = 1, 2) are unknown constants to be determined under certain criterion.

3. Bias and mean square error of proposed estimators T_{11} and T_{12}

Since T_{1u} is a simple linear regression estimator and T_{1m}, T_{2m} are also the simple linear regression type estimators, they are biased for population mean $\bar{Y}$. Since the final estimators T_{11} and T_{12} are convex linear combinations of (T_{1u}, T_{1m}) and (T_{1u}, T_{2m}) respectively, T_{11} and T_{12} are also the biased estimators of $\bar{Y}$ **.** Bias B (.) and mean square error M (.) of the estimators T_{11} and T_{12} are derived up to the first order of approximations and under large sample using the following transformations:

$\bar{y}_u = (1+e_1)\bar{Y}$, $\bar{y}_m = (1+e_2)\bar{Y}$, $\bar{x}_m = (1+e_3)\bar{X}$, $\bar{x}_n = (1+e_4)\bar{X}$, $\bar{z}_m = (1+e_5)\bar{Z}$, $\bar{z}_n = (1+e_6)\bar{Z}$, $\bar{z}_u = (1+e_7)\bar{Z}$, $s_{yz}(u)=(1+e_8)S_{yz}$, $s_{xz}(m)=(1+e_9)S_{xz}$, $s_{xz}(n)=(1+e_{10})S_{xz}$, $s_z^2(u)=(1+e_{11})S_z^2$, $s_z^2(m)=(1+e_{12})S_z^2$, $s_z^2(n)=(1+e_{13})S_z^2$, $b_{yx.z}(m)=\left(1+e_{14}\right)\beta_{yx.z}$, $b_{yz.x}(m)=\left(1+e_{15}\right)\beta_{yz.x}$; Such that $E(e_i)=0$ and $\left|e_j\right|<1 \ \forall \ j=1,2,3,...,15.$

Under the above transformations T_{1u} and T_{jm} (j = 1, 2) take the following forms:

$$T_{1u} = \left(1+e_1\right)\bar{Y} - \left(e_7 + e_7 e_8 - e_7 e_{11}\right)\beta_{yz}\bar{Z} \tag{3}$$

$$T_{1m} = \bar{Y}\left(1+e_2\right)+\beta_{yx.z}\bar{X}\left(e_4-e_3+e_4e_{14}-e_3e_{14}\right)-\beta_{yz.x}\bar{Z}\left(e_5+e_5e_{15}\right) \tag{4}$$

$$T_{2m} = \bar{Y}\left(1+e_2\right)-\beta_{yz.x}\left(e_5+e_5e_{15}\right)+\beta_{yx.z}\bar{X}\left(e_4-e_3+e_4e_{14}-e_3e_{14}\right)$$
$$+\beta_{yz.x}\beta_{xz}\bar{Z}\left(e_5-e_6+e_5e_9-e_5e_{12}-e_6e_{10}+e_6e_{13}+e_5e_{14}-e_6e_{14}\right) \tag{5}$$

Thus, we have the following theorems:

Theorem 3.1: Bias of the sequence of estimators T_{1j} (j = 1, 2) to the first order of approximations is obtained as

$$B(T_{1j}) = \varphi_j B(T_{1u})+(1-\varphi_j)B(T_{jm})\,;\ (j = 1, 2) \tag{6}$$

where

$$B(T_{1u}) = \beta_{yz}\left(\frac{1}{u}-\frac{1}{N}\right)\left(\frac{\alpha_{003}}{S_z^2}-\frac{\alpha_{012}}{S_{yz}}\right) \tag{7}$$

$$B\left(T_{1m}\right) = \delta_1\text{-}\delta_2\text{-}\delta_3 \tag{8}$$

and

$$B\left(T_{2m}\right) = \delta_1\text{-}\delta_2\text{-}\delta_3 + \beta_{xz}\left[\delta_5 - \delta_4 + \beta_{yx.z}\beta_{yz}\left(\frac{1}{m}-\frac{1}{n}\right)\left(\frac{\alpha_{003}}{S_z^2}-\frac{\alpha_{012}}{S_{yz}}\right)\right] \tag{9}$$

where

$$\delta_1 = E\left[\left(\bar{x}_n\text{-}\bar{X}\right)\left(b_{yx.z}(m)\text{-}\beta_{yx.z}\right)\right],$$

$$\delta_2 = E\left[\left(\bar{x}_m\text{-}\bar{X}\right)\left(b_{yx.z}(m)\text{-}\beta_{yx.z}\right)\right],$$

$$\delta_3 = E\left[\left(\bar{z}_n\text{-}\bar{Z}\right)\left(b_{yz.x}(m)\text{-}\beta_{yz.x}\right)\right],$$

$$\delta_4 = E\left[\left(\bar{z}_n\text{-}\bar{Z}\right)\left(b_{yx.z}(m)\text{-}\beta_{yx.z}\right)\right],$$

$$\delta_5 = E\left[\left(\bar{z}_m\text{-}\bar{Z}\right)\left(b_{yx.z}(m)\text{-}\beta_{yx.z}\right)\right],$$

and $\alpha_{rst} = E\left[\left(x\text{-}\bar{X}\right)^r\left(y\text{-}\bar{Y}\right)^s\left(z\text{-}\bar{Z}\right)^t\right]; r \geq 0, s \geq 0, t \geq 0$

Proof: The bias of the sequence of estimators T_{1j} (j = 1, 2) is given by

$$B(T_{1j}) = E\left[T_{1j}\text{-}\bar{Y}\right] = \varphi_j E(T_{1u}\text{-}\bar{Y}) + (1\text{-}\varphi_j)\, E(T_{jm}\text{-}\bar{Y}) \tag{10}$$

$$= \varphi_j B(T_{1u}) + \left(1\text{-}\varphi_j\right) B(T_{jm})$$

where

$$B(T_{1u}) = E\left[T_{1u}\text{-}\bar{Y}\right] \text{ and } B(T_{jm}) = E\left[T_{jm}\text{-}\bar{Y}\right], \left(j = 1, 2\right)$$

Substituting the values of T_{1u}, T_{1m} and T_{2m} from equations (3)–(5) in the equation (10) and taking expectations up to $o(n^{-1})$, we have the expression for the bias of the sequence of estimators T_{1j} (j = 1, 2) as described in equation (6).

Theorem 3.2: Mean square error of the sequence of estimators T_{1j} (j = 1, 2) to the first order of approximations is obtained as

$$M(T_{1j}) = \varphi_j^2 M\left(T_{1u}\right) + \left(1-\varphi_j\right)^2 M\left(T_{jm}\right) + 2\varphi_j\left(1-\varphi_j\right)C_{1j} \quad ; (j = 1, 2) \qquad (11)$$

where

$$M(T_{1u}) = E\left(T_{1u}-\bar{Y}\right)^2 = \left(\frac{1}{u}-\frac{1}{N}\right)\left(1-\rho_{yz}^2\right) S_y^2$$

$$M(T_{1m}) = E\left(T_{1m}-\bar{Y}\right)^2 = \left[\left(\frac{1}{m}-\frac{1}{N}\right)\left(1-2\Delta_1\rho_{yz}+\Delta_1^2\right)+\left(\frac{1}{m}-\frac{1}{n}\right)\left(\Delta_2^2-2\Delta_2\rho_{yx}+2\Delta_1\Delta_2\rho_{xz}\right)\right]S_y^2$$

$$M(T_{2m}) = E\left(T_{2m}-\bar{Y}\right)^2 = \left[\left(\frac{1}{m}-\frac{1}{N}\right)\left(1-2\Delta_1\rho_{yz}+\Delta_1^2\right)+\left(\frac{1}{m}-\frac{1}{n}\right)\left(\Delta_2^2-\Delta_2^2\rho_{xz}^2-2\Delta_2\rho_{yx}\right.\right.$$

$$\left.\left.+2\Delta_2\rho_{xz}\rho_{yz}\right)\right]S_y^2$$

$$C_{11} = E\left[\left(T_{1u}-\bar{Y}\right)\left(T_{1m}-\bar{Y}\right)\right] = -\frac{S_y^2}{N}\left(1-\rho_{yz}^2\right)$$

$$C_{12} = E\left[\left(T_{1u}-\bar{Y}\right)\left(T_{2m}-\bar{Y}\right)\right] = -\frac{S_y^2}{N}\left(1-\rho_{yz}^2\right)$$

$$\text{where } \Delta_1 = \left(\frac{\rho_{yz}-\rho_{yx}\rho_{xz}}{1-\rho_{xz}^2}\right) \text{and } \Delta_2 = \left(\frac{\rho_{yx}-\rho_{yz}\rho_{xz}}{1-\rho_{xz}^2}\right)$$

Proof: It is obvious that mean square errors of the sequence of estimators T_{1j} (j = 1, 2) is given by

$$M(T_{1j}) = E\left[T_{1j}-\bar{Y}\right]^2 = E\left[\varphi_j\left(T_{1u}-\bar{Y}\right)+\left(1-\varphi_j\right)\left(T_{jm}-\bar{Y}\right)\right]^2$$

$$= \varphi_j^2 M(T_{1u}) + \left(1-\varphi_j^2\right)M\left(T_{jm}\right)+2\varphi_j\left(1-\varphi_j\right)E\left[\left(T_{1u}-\bar{Y}\right)\left(T_{jm}-\bar{Y}\right)\right] \; ; (j = 1, 2)$$

Using the expressions given in equations (2)–(5) and taking expectations up to $o(n^{-1})$, we have the expression of mean square error of the sequence of estimators T_{1j} (j = 1, 2) given in equation (11).

4. Minimum mean square error of proposed sequence of estimators T_{1j} (j = 1, 2)

Since mean square error of T_{1j} (j = 1, 2) in equation (11) is a function of unknown constant φ_j, it is minimized with respect to φ_j and subsequently the optimum values of φ_j (j = 1, 2) is obtained as

$$\varphi_{j_{opt}} = \frac{M(T_{jm})-C_{1j}}{M(T_{1u})+M(T_{jm})-2C_{1j}}; \; (j = 1, 2). \tag{12}$$

Now substituting the value of $\varphi_{j\,opt}$ (j = 1, 2) in equation (12), we get the optimum mean square error of T_{1j} (j = 1, 2) as

$$M(T_{1j})_{opt} = \frac{M(T_{1u}).M(T_{jm})-C_{1j}^2}{M(T_{1u})+M(T_{jm})-2C_{1j}}; \; (j = 1, 2) \tag{13}$$

Further, substituting the values of $M(T_{1u})$, $M(T_{jm})$ and C_{1j} (j = 1, 2) in equations (12) and (13), we get the simplified values of $\varphi_{j_{opt}}$ (j = 1, 2) and $M(T_{1j})_{opt}$ (j = 1, 2), which are shown below:

$$\varphi_{1_{opt}} = \frac{\mu_1\left[A_8+\mu_1A_7\right]}{\left[A_1+\mu_1A_6+\mu_1^2A_7\right]} \tag{14}$$

$$M(T_{11})_{opt} = \frac{A_{10}+\mu_1A_{11}+\mu_1^2A_{12}}{A_1+\mu_1A_6+\mu_1^2A_7}\frac{S_y^2}{n} \tag{15}$$

$$\varphi_{2_{opt}} = \frac{\mu_2\left[A_8+\mu_2A_9\right]}{\left[A_1+\mu_2A_6+\mu_2^2A_9\right]} \tag{16}$$

$$M(T_{12})_{opt} = \frac{A_{10}+\mu_2A_{13}+\mu_2^2A_{14}}{A_1+\mu_2A_6+\mu_2^2A_9}\frac{S_y^2}{n} \tag{17}$$

where

$$A_1 = 1-\rho_{yz}^2, \quad A_2 = 1-2\Delta_1\rho_{yz}+\Delta_1^2, \quad A_3 = \Delta_2^2-2\Delta_2\rho_{yx}+2\Delta_1\Delta_2\rho_{xz},$$

$$A_4 = \Delta_2^2-\Delta_2^2\rho_{xz}^2-2\Delta_2\rho_{yx}+2\Delta_2\rho_{xz}\rho_{yz}, \quad A_5 = A_1-A_2, \quad A_6 = -(1-f)A_5,$$

$$A_7 = A_3\text{-}fA_5, \quad A_8 = A_2+fA_5, \quad A_9 = A_4\text{-}fA_5, \quad A_{10} = (1\text{-}f)A_1A_2,$$

$$A_{11} = (A_3\text{-}f^2A_5)A_1, \quad A_{12} = \text{-}fA_1A_7, \quad A_{13} = (A_4\text{-}f^2A_5)A_1,$$

$$A_{14} = \text{-}fA_1A_9 \quad \text{and } f = \frac{n}{N}.$$

5. Optimum replacement policy

To determine the optimum values of μ_j (j =1, 2) (fraction of samples to be taken afresh at second occasion) so that population mean $\overline{Y}$ may be estimated with maximum precision, we minimize mean square errors of T_{1j} (j =1, 2) given in equations (15) and (17) respectively with respect to μ_j (j =1, 2), which result in quadratic equations in μ_j (j =1, 2) and respective solutions of μ_j, say $\hat{\mu}_j$ (j =1, 2) are given below:

$$Q_1\mu_1^2 + 2Q_2\mu_1 + Q_3 = 0 \tag{18}$$

$$\hat{\mu}_1 = \frac{\text{-}Q_2 \pm \sqrt{Q_2^2\text{-}Q_1Q_3}}{Q_1} \tag{19}$$

$$Q_4\mu_2^2 + 2Q_5\mu_2 + Q_6 = 0 \tag{20}$$

$$\hat{\mu}_2 = \frac{\text{-}Q_5 \pm \sqrt{Q_5^2\text{-}Q_4Q_6}}{Q_4} \tag{21}$$

where $Q_1 = A_6A_{12}\text{-}A_7A_{11}$, $Q_2 = A_1A_{12}\text{-}A_7A_{10}$, $Q_3 = A_1A_{11}\text{-}A_6A_{10}$, $Q_4 = A_6A_{14}\text{-}A_9A_{13}$, $Q_5 = A_1A_{14}\text{-}A_9A_{10}$ and $Q_6 = A_3A_{13}\text{-}A_6A_{10}$.

From equations (18) and (20), it is obvious that real values of μ_j (j =1, 2) exist if the quantities under square roots are greater than or equal to zero. For any combination of correlations ρ_{yx}, ρ_{xz} and ρ_{yz}, which satisfy the conditions of real solutions, two real values of $\hat{\mu}_j$ (j =1, 2) are possible. Hence, while choosing the values of $\hat{\mu}_j$, it should be remembered that $0 \le \hat{\mu}_j \le 1$, all the other values of μ_j (j =1, 2) are inadmissible. Substituting the admissible values of $\hat{\mu}_j$ say $\mu_j^{(0)}$ (j =1, 2) from equations (19) and (21) into equations (15) and (17) respectively, we have the optimum values of mean square errors of T_j (j =1, 2), which are shown below:

$$M(T_{11})_{opt} = \frac{A_{10}+\mu_1^{(0)}A_{11}+\mu_1^{(0)2}A_{12}}{A_1+\mu_1^{(0)}A_6+\mu_1^{(0)2}A_7}\frac{S_y^2}{n} \quad (22)$$

$$M(T_{12})_{opt} = \frac{A_{10}+\mu_2^{(0)}A_{13}+\mu_2^{(0)2}A_{14}}{A_1+\mu_2^{(0)}A_6+\mu_2^{(0)2}A_9}\frac{S_y^2}{n} \quad (23)$$

6. Efficiency comparison

The percent relative efficiency of the sequence of estimators $T_{1j}(j = 1, 2)$ with respect to (i) $\overline{y}_n$, when there is no matching and (ii) $\hat{\overline{Y}}=\varphi^*\overline{y}_u+(1-\varphi^*)\overline{y}'_m$, when no auxiliary information is used at any occasion, where $\overline{y}'_m=\overline{y}_m+\beta_{yx}\left(\overline{x}_n-\overline{x}_m\right)$, have been obtained for different choices of ρ_{yx}, ρ_{yz} and ρ_{xz}. Since $\overline{y}_n$ and $\hat{\overline{Y}}$ are unbiased estimators of $\overline{Y}$, following Sukhatme et al. (1984), the variance of $\overline{y}_n$ and optimum variance of $\hat{\overline{Y}}$ are given by

$$V(\overline{y}_n) = \left(\frac{1}{n}-\frac{1}{N}\right)S_y^2$$

$$V\left(\hat{\overline{Y}}\right)_{opt^*} = \left[1+\sqrt{1-\rho_{yx}^2}\right]\frac{S_y^2}{2n}-\frac{S_y^2}{N}$$

For N=5000, n=500 and different choices of ρ_{yx}, ρ_{yz} and ρ_{xz}, Tables 1–2 give the optimum values of μ_j (j = 1, 2) and percent relative efficiencies $E_j^{(1)}$ and $E_j^{(2)}$ of $T_{1j}(j = 1, 2)$ with respect to $\overline{y}_n$ and $\hat{\overline{Y}}$ respectively, where

$$E_j^{(1)} = \frac{V(\overline{y}_n)}{M\left(T_{1j}^0\right)_{opt}}\times 100 \text{ and } E_j^{(2)} = \frac{V\left(\hat{\overline{Y}}\right)_{opt^*}}{M\left(T_{1j}^0\right)_{opt}}\times 100 \quad , (j = 1, 2).$$

The percent relative efficiencies are also demonstrated by means of pictorial representation for some choices of correlations.

Table 1. Optimum values of μ_1 and percent relative efficiencies of T_{11} with respect to $\overline{y}_n$ and $\hat{\hat{\overline{Y}}}$

ρ_{yx}		0.3			0.5			0.7			0.9		
ρ_{xz}	ρ_{yz}	$\mu_1^{(o)}$	$E_1^{(1)}$	$E_1^{(2)}$	$\mu_1^{(o)}$	$E_1^{(1)}$	$E_1^{(2)}$	$\mu_1^{(o)}$	$E_1^{(1)}$	$E_1^{(2)}$	$\mu_1^{(o)}$	$E_1^{(1)}$	$E_1^{(2)}$
0.3	0.3	0.5471	111.42	108.57	0.5669	116.28	107.62	0.6088	126.74	106.61	0.7125	153.47	105.37
	0.5	0.5447	134.47	131.03	0.5636	140.08	129.66	0.6076	153.40	129.04	0.7287	191.41	131.42
	0.7	0.5429	196.95	191.91	0.5638	206.12	190.77	0.6236	232.78	195.81	*	-	-
	0.9	0.5415	527.01	513.53	0.5827	575.66	532.82	*	-	-	*	-	-
0.5	0.3	0.6186	110.67	107.84	0.6344	114.55	106.02	0.6713	123.71	104.06	0.7727	149.70	102.79
	0.5	0.6158	133.46	130.04	0.6267	136.69	126.52	0.6581	146.11	122.91	0.7440	172.55	118.47
	0.7	0.6160	196.35	191.32	0.6212	198.62	183.83	0.6524	212.33	178.61	0.7586	260.53	178.88
	0.9	0.6319	545.70	531.73	0.6171	528.30	488.98	0.6705	591.59	497.64	*	-	-
0.7	0.3	0.7330	110.17	107.35	0.7453	113.17	104.74	0.7802	121.81	102.46	*	-	-
	0.5	0.7323	133.46	130.04	0.7359	134.53	124.52	0.7577	141.02	118.62	0.8317	163.58	112.32
	0.7	0.7419	200.45	195.32	0.7319	196.08	181.49	0.7443	201.51	169.51	0.8000	226.22	155.32
	0.9	*	-	-	0.7447	541.37	501.07	0.7353	530.38	446.15	0.8218	633.64	435.06
0.9	0.3	0.8968	109.92	107.11	0.9060	112.19	103.84	*	-	-	*	-	-
	0.5	0.9011	134.64	131.20	0.8971	133.46	123.53	0.9114	137.73	115.86	*	-	-
	0.7	*	-	-	0.9016	198.23	183.48	0.8980	196.66	165.43	0.9313	211.30	145.08
	0.9	*	-	-	*	-	-	0.9071	538.64	453.10	0.9032	533.99	366.64

* *indicates that* $\mu_1^{(0)}$ *does not exist.*

Table 2. Optimum values of μ_2 and percent relative efficiencies of T_{12} with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$

ρ_{yx}		0.3			0.5			0.7			0.9		
ρ_{xz}	ρ_{yz}	$\mu_2^{(o)}$	$E_2^{(1)}$	$E_2^{(2)}$	$\mu_2^{(o)}$	$E_2^{(1)}$	$E_2^{(2)}$	$\mu_2^{(o)}$	$E_2^{(1)}$	$E_2^{(2)}$	$\mu_2^{(o)}$	$E_2^{(1)}$	$E_2^{(2)}$
0.3	0.3	0.5505	111.25	108.41	0.5679	115.52	106.92	0.6037	124.41	104.65	0.6829	144.63	**
	0.5	0.5484	134.35	130.91	0.5650	139.30	128.93	0.6026	150.64	126.72	0.6937	178.91	122.84
	0.7	0.5468	196.86	191.82	0.5652	204.94	189.69	0.6158	227.43	191.31	0.8037	315.71	216.77
	0.9	0.5455	526.94	513.45	0.5816	569.49	527.10	0.8542	914.76	769.49	*	-	-
0.5	0.3	0.6571	110.35	107.53	0.6662	112.57	104.19	0.6853	117.29	**	0.7230	126.70	**
	0.5	0.6554	133.40	129.99	0.6618	135.30	125.23	0.6788	140.36	118.07	0.7145	151.13	103.76
	0.7	0.6555	196.24	191.22	0.6586	197.58	182.87	0.6759	205.13	172.55	0.7190	224.29	154.00
	0.9	0.6648	537.55	523.79	0.6562	527.50	488.24	0.6849	561.33	472.18	0.8713	791.85	543.69
0.7	0.3	0.9783	109.89	101.71	0.9784	109.91	**	0.9785	109.94	**	0.9783	133.33	129.92
	0.5	0.9783	133.33	123.41	0.9784	133.35	112.17	0.9785	133.38	**	0.9783	196.09	191.07
	0.7	0.9783	196.07	181.48	0.9783	196.09	164.95	0.9784	196.13	134.67	0.9786	526.60	513.12
	0.9	0.9783	526.35	487.18	0.9783	526.32	442.74	0.9785	526.50	361.50	*	-	-

** indicates that $\mu_2^{(0)}$ does not exist and '**' indicates no gain.*

Figure 1. The PRE of T_{11} w. r. t. $\bar{y}_n$ for $\rho_{xz} = 0.5$ and different values of ρ_{yz} and ρ_{yx}

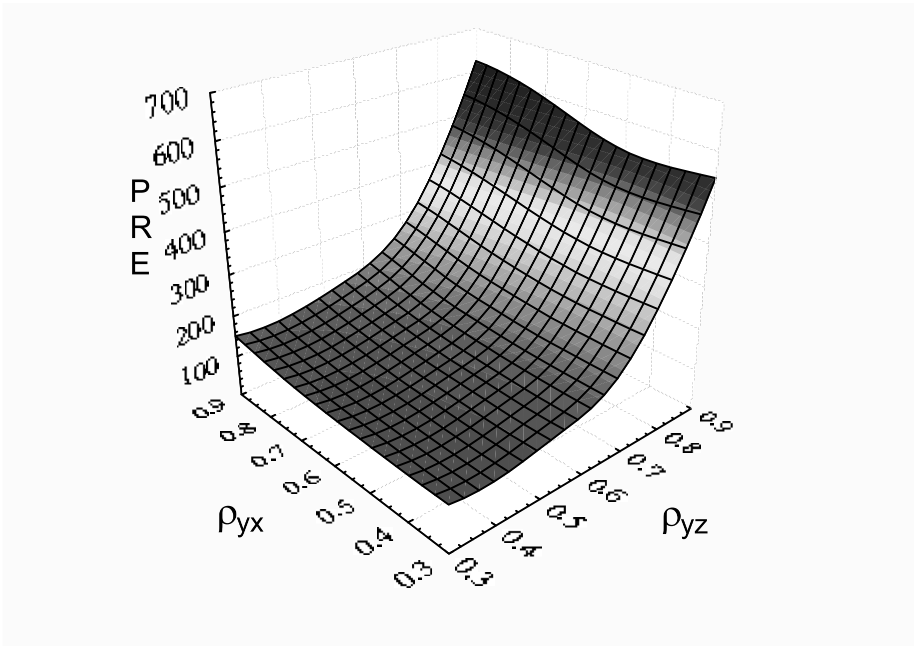
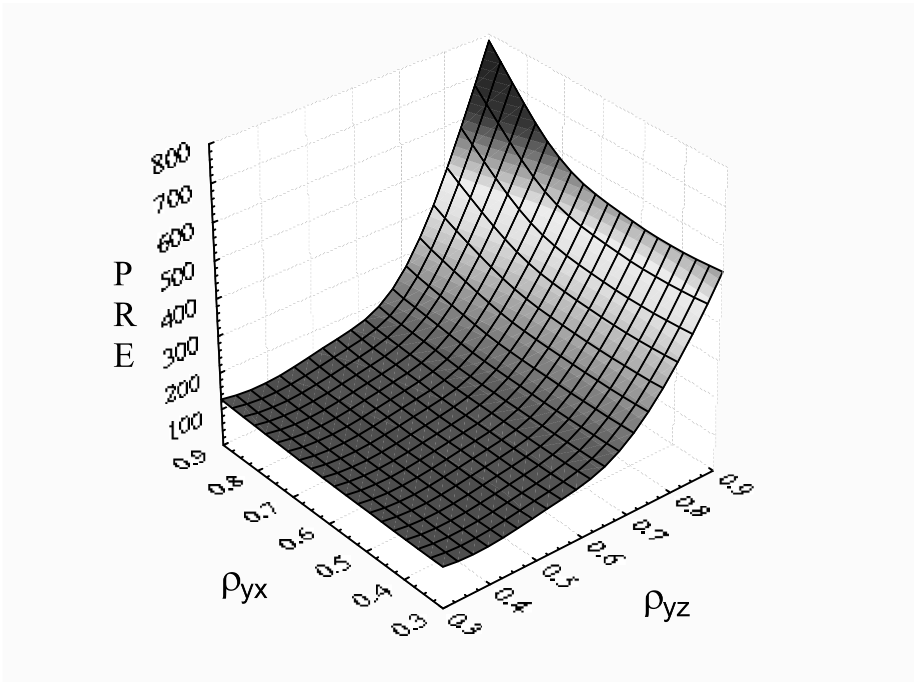


Figure 2. The PRE of T_{12} w. r. t. $\overline{y}_n$ for $\rho_{xz} = 0.5$ and different values of ρ_{yz} and ρ_{yx}



7. Conclusion

The following conclusions can be read out from Tables 1–2:

(1) From Table 1 it is noticed that

(a) For fixed values of ρ_{xz} and $\rho_{yz,}$ the values of $E_1^{(1)}$, $E_1^{(2)}$ and $\mu_1^{(0)}$ are increasing with the increasing values of $\rho_{yx.}$ This behaviour is in agreement with Sukhatme et al. (1984) results, which explains that the more the value of ρyx, more the fraction of fresh sample is required at the current occasion.

(b) For the smaller values of ρ_{xz} and $\rho_{yx,}$ the values of $E_1^{(1)}$ and $E_1^{(2)}$ are increasing while the values of $\mu_1^{(0)}$ are decreasing with the increasing values of ρ_{yz} . For the larger values of ρ_{xz} and ρ_{yx} , the values of $E_1^{(1)}$ and $E_1^{(2)}$ are increasing with the increasing values of ρ_{yz} while no definite patterns are noticed for the variations in the values of $\mu_1^{(0)}$ in similar situations. This behaviour is highly desirable in terms of percent relative efficiencies, since it concludes that if highly correlated auxiliary character is available, it pays in terms of enhance precision of estimates.

(c) For fixed values of ρ_{yz} and ρ_{yx}, the values of $E_1^{(1)}$ and $E_1^{(2)}$ are decreasing while the values of $\mu_1^{(0)}$ are increasing with the increasing values of ρ_{xz}. This indicates that behaviour of the correlation ρ_{xz} at first occasion is sensitive in terms of percent relative efficiencies and amount of fresh sample to be replaced at current occasion.

(2) From Table 2 it is observed that

(a) For fixed values of ρ_{yz} and ρ_{xz}, the values of $\mu_2^{(0)}$ are increasing and the values of $E_2^{(1)}$ are decreasing while the values of $E_2^{(2)}$ do not follow any definite pattern with the increasing values of ρ_{yx}.

(b) For fixed values of ρ_{xz} and ρ_{yx}, the values of $E_2^{(1)}$ and $E_2^{(2)}$ are increasing with the increasing values of ρ_{yz} while no definite patterns are observed in the values of $\mu_2^{(0)}$.

(c) For fixed values of ρ_{yz} and ρ_{yx}, the values of $\mu_{21}^{(0)}$ increase while $E_2^{(1)}$ and $E_2^{(2)}$ do not show any predictable pattern when the values of ρ_{xz} are increased. Nevertheless, overall gain is observed in terms of percent relative efficiencies.

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